STRUCTURE OF A COMBINATION DISCONTINUITY IN GAS SUSPENSIONS IN THE PRESENCE OF RANDOM PRESSURE FROM PARTICLES

A. V. Fedorov

UDC 532.529

Fedorov and Fomin [1] derived the conditions for the existence of a combination discontinuity (CMD) in a gas suspension, i.e., realization of the situation in which certain flow parameters become discontinuous on a line in the space (x, t) in the absence of particulate mass flow. The same study also presented a brief overview of investigations in the mechanics of heterogeneous media and gasdynamics in channels with an abruptly changing geometry. Below, we study the structure of a CMD in gas suspensions with allowance for radiation pressure from particles. Here, as in [1], the phrase "structure of a CMD" is taken to imply the existence of a flow of gas which encounters a cloud of particles and is slowed or accelerated in it.

A finite cloud of small particles dispersed in a unidimensional space will be examined. Let a gas flow into this cloud. The parameters of the gas undergo discontinuities at the edge of the cloud, with the gas then passing through a collection of particles with a variable concentration. The gas leaves the cloud a certain finite distance from the place where it entered. We will study the given flow on the basis of a model from the mechanics of heterogeneous media. The equations describing the CMD are taken from [1] for the case when random motion of the particles is ignored. In the given variant, these conditions appear as follows when written in the corresponding coordinate system

$$\begin{gathered}[\rho_i u_i] = 0,\ i = 1,2,\ p_2 = p_\sigma m_2,\ p_\sigma = \text{const},\\ [c_1 u_1 + m_1 p] = [m_1]p',\ [c_2 u_2 + m_2 p + p_2] = [m_2]p',\\ m_1 + m_2 = 1,\ p = a^2\rho,\ c_i = \rho_i u_i,\ c_2 = 0,\ u_2 = 0,\ u_1 = u.\end{gathered} \tag{1}$$

Here, u_i and m_i are the velocity and volume concentration of the i-th phase; i = 1 denotes the gas; i = 2 denotes the particles; p is the pressure of the gas; p_2 is the random motion of the particles $p' = \lim_{\varepsilon\to 0}\int_{x_0(t)-\varepsilon}^{x_0(t)+\varepsilon} p(x,t)\,\delta(x - x_0(t))\,dx$ is the pressure acting along the front of the CMD. The equations of mass and momentum conservation for each phase are written as follows in the coordinate system $\xi = x - Dt$:

$$\begin{gathered}c_1\dot{u} + m_1\dot{p} = -m_2 f,\ p = a^2\rho,\ c_1 = \rho m_1 u = \rho_0 u_0,\\ \dot{p}_2 + m_2\dot{p} = m_2 f,\ m_1 + m_2 = 1,\ f = \rho_2 c_D \operatorname{Re} u/24\tau_{st}\, m_2,\ \tau_{st} = 2r^2\rho_{22}/9\mu\end{gathered} \tag{2}$$

(the dot denotes the derivatives $d/d\xi$). Then the formulation of the problem of determining the structure of a CMD in a gas suspension reduces to the boundary-value problem: find functions $(\rho, u, p, m_2, p_2) = \Phi$, and constant L in the region $R_\xi\{R_\xi{:}\ \xi \in (0, L)\}$, which satisfy Eqs. (2) and boundary conditions (1) in this region at $\xi = 0$, as well as the condition

$$M = M_f \text{ at } \quad \xi = L. \tag{1'}$$

We reduce problem (1), (1'), (2) to the study of a boundary-value problem for an ordinary differential equation. Equations (2) have the momentum conservation law for the mixture as a whole: $p + c_1 u + p_2 = c_2 = p_0 + c_1 u_0$. Using a corollary of the continuity equation for the gas written in the form $p = a^2 c_1/m_1 u = c_2 - c_1 u - p_\sigma m_2$, we find $dm_2/d\xi = (du/d\xi)(m_1(p - c_1 u)/(p + m_1 p_\sigma))$, $\dot{p} = -c_1\dot{u} - p_\sigma \dot{m}_2$. It then follows from the last result that $\dot{p} = -\dot{u}x(c_1 up + pp_\sigma m_1)/u(p + m_1)$. Having inserted this expression into the first equation of (2), we obtain

$$B_1(M)\frac{M^2 - \tilde{a}^2}{M^2}\dot{M} = -\frac{\rho_2}{\tau_{st} c_1}\frac{c_D \operatorname{Re}}{24} M, \tag{3}$$

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 5, pp. 36-41, September-October, 1992. Original article submitted July 4, 1991.

TABLE 1

m_1	L_m	$L_m\tilde{m}_2$	L_m
0,9967	$\tilde{M}$	1,46	442,0
0,9970	0,302	3,78	1259
0,99927	0,120	32,1	4585,9

TABLE 2

M_0	$\tilde{M}$	$\bar{m}_2 L$
0,1	0,133	25,7
0,2	0,251	6,05
0,3	0,401	1,68
0,4	0,554	0,538
0,5	0,802	0,057

where $B_1(M) = (m_2P + m_1)/(P + m_1)$; $P = p/p_\sigma$; $\tilde{a}^2 = m_1/(m_1 + m_2P)$.

Let $p_\sigma \to 0$, so that $P \to \infty$, $B_1 \to m_2$, $\tilde{a} \to 0$ and (3) becomes the equation $c_1\dot{u} = m_2f$. The same equation was used in [1] to describe a CMD in a mixture without random particle pressure.

We will rewrite the boundary conditions in problem (1), (1'), (2) in terms of the function M, which means that we obtain the Hugoniot curve for the CMD (with $\xi = 0$):

$$H(M, M_0) = m_1M_0M^2 + m_1M\left(m_2p_\sigma - \left(1 + M_0^2\right)\right) + M_0 \equiv AM^2 + BM + C = 0 \tag{4}$$

($p_\sigma = p_\sigma/\rho_0a^2$, $M_0 = u_0/a$). The two branches of the solution of Eq. (4) have the form $M^\pm = (-B \pm \sqrt{\tilde{D}})/2A$ (A, B, and C are coefficients of the quadratic trinomial, while the discriminant $\tilde{D} = m_1[(m_2p_\sigma - \alpha)^2 - (m_2p_\sigma - \alpha)^2m_2 - 4M_0^2]$, $\alpha = 1 + M_0^2$). The function $\tilde{D}(m_1)/m_1$ is a third-degree polynomial relative to m_1. Its three roots, found for $p_\sigma \gg 1$, have the form $m_1^\pm \approx 1 - (1 \pm M_0^2)^2/p_\sigma$, $m_1^0 \approx 4M_0^2/p_\sigma$. It follows from this that, since $\tilde{D}(m_1 = 1) > 0$, the roots of Eq. (4) exist when

$$m_1 \in (m_1^0, m_1^+) = I_1, \quad m_1 \in (m_1^-, 1) = I_2, \; m_* \equiv m_1^-. \tag{5}$$

Here, we easily see that in the interval I_1 the values of $M^\pm$ are negative, i.e., do not yield a physically correct solution. We will take a closer look at $M^\pm$ in the interval I_2. Let $\tilde{D} = 0$, so that $M^\pm = M_* = m_1^{-1/2}$ and, at the point of rotation when $p_\sigma \gg 1$, $M_* \approx 1 + m_2/2 > 1$. In the same approximation, $\tilde{a} \sim 1 - m_2P$, from which $M_*/a \approx 1 + m_2(0.5 + P) > 1$. Also, at $m_1 = 1$, $M^\pm = M_0^{-1}$, M_0. Choosing $M_0 < 1$, we find that only the quantitative characteristics change, due to the continuity of the function $M^\pm(m_1)$. Here, at a certain $m_{**} \in (m_*, 1)$, $M_-/\tilde{a} = 1$, i.e., the curve of final states beyond the CMD has properties similar to those which exist when $p_\sigma = 0$ [1]. This situation remains in force when $M_0 > 1$ and leads to the following.

<u>THEOREM</u>. A positive solution to Eq. (4), determining the parameters of the gas flow beyond the front of the CMD, exists in the region $m_1 \in I_2$ in the form of upper (supersonic) and lower (mixed) branches. Here, the parameters beyond the CMD front are supersonic on the lower branch when $m_1 \in (m_*, m_{**}) = I_{22}$, are subsonic when $m_1 \in (m_{**}, 1) = I_{21}$, and are sonic ($M_-/\tilde{\alpha} = 1$) when $m_1 = m_{**}$.

Thus, it has been shown that when $1 \le m_1 \le m_* \approx 1 - (1 - M_0^2)^2/p_\sigma$, conditions (4) for a CMD make it possible to find the value of the function $M = \tilde{M}(m_1)$ beyond the front of the CMD. Also adding the condition $M = M_f$ on the free boundary $\xi = L$, we see that problem (1)-(2) is reduced to boundary-value problem (3), (1'), (4). Here, the function m_1 is found from the conservation integrals as $m_1 = m_1(M)$.

Let us examine certain aspects of the qualitative behavior of problem (3), (1'), (4). Let $M_0 < 1$, $m_1 \in (m_{**}, 1)$. Then the value of $\tilde{M}$ belonging to the lower branch of the solution of (4) is less than 1. As a result, $\dot{M} > 0$, and a subsonic gas flow with friction is accelerated to the speed of sound. Here, m_1 is found from the equation $m_1^2Mp_\sigma + m_1M(1 + M_0^2 - M_0^2M - p_\sigma) - M_0 = 0$. In this case, we also determine the length of the cloud of particles L and the point at which $M = M_f$. It should be noted that there is an infinite velocity gradient at the flow point where $M_f = 1$.

Similar motion of a gas was studied in [2] in a description of flow in a tube with allowance for friction and heat input. In particular, the author introduced the notion of the maximum corrected length of tube for a certain initial state.

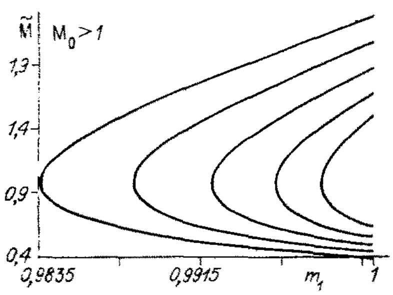

Fig. 1

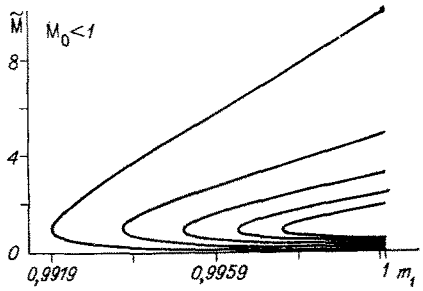

Fig. 2

Following [2], we will determine the maximum length of a cloud of particles L_m as the size for which $M_f = 1$ on the rear edge. In this case, if $M_0 < 1$, $m_1 \in (m_{**}, 1) = I_{21}$, then a subsonic regime is realized on the approach to the rear boundary of the cloud at $L < L_m$ and a sonic regime is realized at $L = L_m$.

Generally speaking, it is also possible to construct a formal solution for $M_0 < 1$, $m_1 \in I_{22}$, when $\tilde{M} > 1$ and belongs to the lower branch. However, here it is necessary to make an artificial assumption on the functioning of the leading edge of the CMD as a de Laval nozzle. The latter converts a continuous subsonic flow into a subcritical flow. By virtue of (3), $\dot{M} < 0$, and the flow is slowed to subsonic speed (or to supersonic speed if $L < L_m$). Special attention should be given to the case $L > L_m$, since it is not realized under steady-state conditions.

At $M_0 < 1$, $m_1 \in I_2$, but $\tilde{M} > 1$, i.e., $\tilde{M}$ belongs to the upper branch of the CMD. The solution $M = M(x)$ is described by a decreasing function in this case. We introduce L_* (the cloud length at which the final value is $M = M_*$). Then when $L < L_*$, $M_f > M_*$, and when $L = L_*$, $M = M_*$. However, here – as before – the problem is to verify the transition from $M_0 < 1$ on the left edge of the CMD to $\tilde{M} > 1$ on its leading edge. This transition is unstable.

Let $M_0 > 1$. We have subsonic flow at the exit from the cloud when $m_1 \in I_{21}$, $L < L_m$ and sonic flow when $L = L_m$. If $m_1 \in I_{22}$, beginning with $M_0 > 1$ at the entry to the confined space (gas suspension) a supersonic flow undergoes deceleration to $\tilde{M} \in (1, M_*)$ ($M_* \equiv M^{\pm}(m_*)$). At $L < L_m$ the flow is transformed at the exit from the cloud into a supersonic flow of lower velocity, but at $L = L_m$ it is changed to a sonic flow. If $m_1 \in (m_*, 1)$ and $\tilde{M}$ belongs to the upper branch, then L_* is determined as above. When $L < L_*$, the final state $M_f \geq M_*$.

Let us examine an approximate analytic solution to the problem of the structure of flow in a CMD. Let $M_0 = 0.1$, $\rho_{22} = 2 \cdot 10^3$ kg/m³, $a_\sigma = 100$ m/sec, $p_\sigma = \rho_{22} a_\sigma^2 = 2 \cdot 10^7$ N/m², $\rho_0 = \rho_{11.0} = 1$ kg/m³, $a = 300$ m/sec, $p_0 = \rho_0 a^2 = 9 \cdot 10^4$ N/m², $p_\sigma / p_0 = 2.22 \cdot 10^2 \gg 1$, $m_* \sim 0.9967$. We choose $m_1 \in I_{21}$. Since the solution for m_1 and m_2 changes within narrow limits, we can put $m_1 \sim 1$, $m_2 \sim m_{20}$ and for $c_D\mathrm{Re} = 24$ obtain $m_2 x = \ln(\tilde{M}/M) + (\tilde{M}^{-2} - M^{-2})/2$.

Table 1 shows data on L_m with $M_0 = 0.1$ for different m_1 at the inlet to a CMD. It is evident that allowance for random particle motion leads to a substantial (compared to [1]) increase in the length of the cloud that can propagate steadily into the gas flow. This length is greater, the lower the concentration of particles.

It is interesting to examine the effect of initial flow velocity on L_m with a fixed volume concentration m_1 at the entry to the cloud. The data in Table 2 (for $m_1 = 0.999$) shows a decrease in the corrected length of the cloud L_m with an increase in M_0. This is to be expected, due to the increase in M_0.

Let us discuss the results in the general case $m_1 \nsim 1$, $m_2 \neq m_{20}$. When performing numerical calculations, in addition to the above data we used the following values for the constants: $\mu = 2 \cdot 10^{-5}$, $\tau_{st} = 2R^2 \rho_{22} / 9\mu$, $\mu_0 = p_0 x_0 / a$, $x_0 = a\tau_{st}$ (the zero subscript denotes parameters used in obtaining dimensionless values).

Figures 1 and 2 show the relations for the Mach number behind the CMD front when $p_\sigma = 10^2$, $M_0 > 1$, and $M_0 < 1$, respectively. It can be seen that a supersonic incident flow is characterized by greater contraction of the state curve beyond the front. The boundary point m_*, defining the region of existence of real states beyond the CMD, turned out to be higher in the asymptotic representation $\bar{m}_1 = 1 - (1 - M_0^2)^2 / p_\sigma$. It follows from this that the limiting value of volumetric particle concentration at which steady flow exists decreases with an increase in particle velocity.

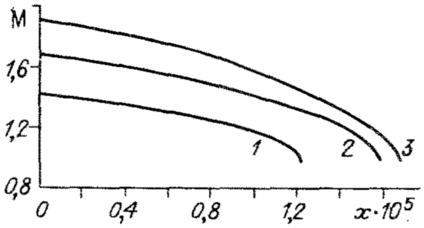

Fig. 3

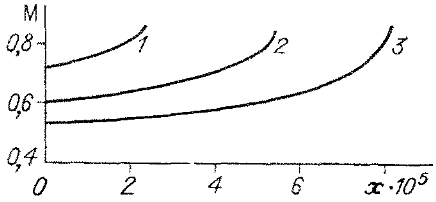

Fig. 4

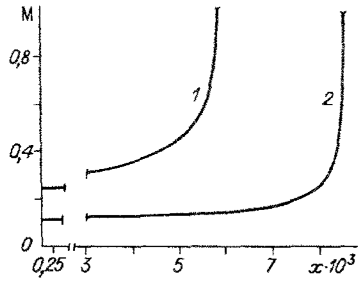

Fig. 5

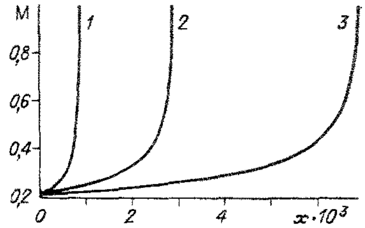

Fig. 6

Let us attempt to provide a physical interpretation for the upper and lower branches of the solution $\tilde{M}(m_1)$. Let $M_0 > 1$, i.e., suppose that a supersonic flow enters a dust-bearing space. The cross section within which the gas flows decreases. By making a gasdynamic analogy with pipe flow, we can establish that flow velocity also decreases in this case. The supersonic flow decelerates and the subsequent motion of the mixture takes place in accordance with (4). Illustrative of this type of flow is the Mach-number distribution along the cloud for $p_c = 10$, $R = 10^{-4}$ m (particle radius), $m_{10} = 0.9758$ (Fig. 3, lines 1-3 for $M_0 = 1.7$, 1.9, 2.1). Delayed at the edge of the CMD, the gas continues to slow to the velocity $u_* = 1/\sqrt{m_{1*}}$ at the exit from the cloud where $\xi = L_* - 0$, while the concentration of gas decreases to $m_1 = m_*$. If $m_1 = m_{10}$ and we take the Cauchy data for the lower branch, then the gas in the shock wave attached to the edge of the CMD slows to $\tilde{M} < 1$ before accelerating. It should be noted that a qualitatively similar type of flowing gas suspension was seen in [3]. The author established a limiting particle concentration $m_2 = 0.01$ for bronze and organic glass (the analog of the quantity m_* in our model). At this concentration, individual shock waves near the particles merge and form an attached suspended shock ahead of the particle cloud. If $M_f = 0.82$ (as shown in Fig. 4, where the notation conforms to Fig. 3), then the corrected length of the cloud increases with an increase in the initial velocity of the gas. In this case, the volume concentration of gas decreases to m_{**} if $M_f = 1$.

We studied the effect of random particle velocity a_σ on the flow pattern in the relaxation zone. It was found that an increase in a_σ leads to a decrease in cloud length L_m (Fig. 5, line 1 for $M_0 = 0.1$, $m_{10} = 0.9919$, $\tilde{M} = 0.1098$, $p_\sigma = 10^2$, line 2 for $p_\sigma = 10$, $m_{10} = 0.9943$, $\tilde{M} = 0.240$). In fact, an increase in p_σ is accompanied by a decrease in the limiting particle concentration m_{2*} and the size of these particles. This in turn results in a decrease in dissipation of momentum on the particles, i.e., the gas accelerates to the speed of sound more rapidly. A similar effect is seen from an increase in particle radius R: cloud width decreases with an increase in radius. This can be attributed to the fact that there is an increase in acceleration of the flow in a cloud with large values of particle radius. This can readily be seen from the estimate $a \approx a_0(1 + 6\mathrm{Re}^{2/3})$, where a is the acceleration of the gas and a_0 is its characteristic value. For the Klyachko formula used as an example, we took a drag coefficient $c_D = 24(1 + 6\mathrm{Re}^{2/3})/\mathrm{Re}$. This situation is illustrated by the data in Fig. 6 ($M_0 = 0.1$, $\tilde{M} = 0.2089$, $p_\sigma = 10$, $m_{10} = 0.9514$, lines 1-3 for $R = 10^{-4}$, 10^{-5}, 10^{-6}). Figure 7 shows distributions of gas velocity in a cloud for different values of the Stokes drag coefficient c_D. Also shown is data from [4] (lines 1, 2). The considerable length of the cloud in the case of the Stokes law for flow about a particle is due to its lesser acceleration during the flow process — as was explained above in our examination of the effect of variation of the radius.

It is interesting to examine the distribution of particle concentration in the cloud depicted in Fig. 8 ($M_0 = 0.1$, $\tilde{M} = 0.1098$, $p_\sigma = 10$) for $R = 10^{-4}$, 10^{-5}, 10^{-6} (lines 1-3). Flow in this case is similar to the gas flow in the subsonic region of a de Laval nozzle.

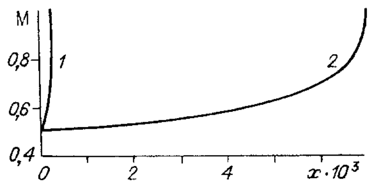

Fig. 7

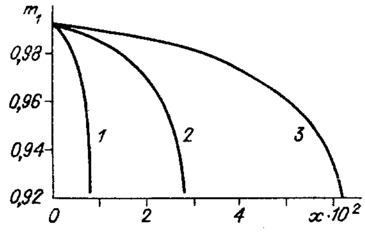

Fig. 8

In fact, the subsonic flow beyond the CMD at $\tilde{M} < 1$ is accelerated by the flow of gas into the convergent section, since the concentration of particles increases by the end of the cloud. There is a fairly abrupt change in the through section of the gas at $R = 10^{-4}$ m, with a subsequent decrease in particle radius leading to smoothing of the particle concentration profile.

There is a different distribution of particles in the cloud if the incoming flow is supersonic ($m_{10} = 0.9757$, $R = 10^{-4}$, $p_\sigma = 10$, $M_0 = 1.7$, $\tilde{M} = 0.72$, $M_0 = 1.9$, $\tilde{M} = 0.6$, $M_0 = 2.1$, $\tilde{M} = 0.53$) and the flow behind the edge is subsonic. This situation actually corresponds to an attached shock wave. In the given case, the particles in the cloud undergo a fair degree of consolidation. Meanwhile, with an increase in M_0, the cloud grows and the particles are compacted toward the cloud's end.

Thus, a mathematical model has been proposed to describe the structure of the CMD in a gas suspension with allowance for the random motion of the particles. Classifications have been given for stable and unstable types of steady flows of gas suspensions in a CMD, and corresponding numerical examples have been provided. The empirically observed fact of the existence of a flow with an attached shock wave on a particle cloud was cited as an analog of one of the possible regimes.

LITERATURE CITED

1. A. V. Fedorov and V. M. Fomin, Toward a Theory of Combination Discontinuities in Gas Suspensions. Physical Gasdynamics of Regulating Media [in Russian], Nauka, Novosibirsk (1990).
2. G. G. Chernyi, Gasdynamics [in Russian], Nauka, Moscow (1988).
3. V. M. Boiko, Investigation of the Dynamics of the Acceleration, Destruction, and Ignition of Particles Behind Shock Waves by Laser Visualization Methods. Author's Abstract of Physicomathematical Sciences Candidate Dissertation, Novosibirsk (1984).
4. G. V. Tsiklauri, V. S. Danilin, and L. I. Seleznev, Adiabatic Two-Phase Flows [in Russian], Atomizdat, Moscow (1973).